Differentiate the following(10-14) functions w. r .t . x :

10. (i) $x\tan^{-1}x$ ii) $x\cos^{-1}\sqrt{x}$ iii) $\frac{\sin^{-1}x}{x}$

$$\frac{d}{dx}uv = v\frac{d}{dx}u + u\frac{d}{dx}v$$

$$\frac{dy}{dx} = \frac{x}{1+x^2} + \tan^{-1}x$$ Ans

(i) $x\tan^{-1}x$

$$y = x\tan^{-1}x$$

$$\frac{dy}{dx} = x\frac{d}{dx}\tan^{-1}x + \tan^{-1}x\frac{d}{dx}x$$

$$= x\times\frac{1}{1+x^2} + \tan^{-1}x\times 1$$

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(ii) $x \cos^{-1}\sqrt{x}$

$$y = x \cos^{-1}\sqrt{x}$$

$$\frac{dy}{dx} = x \frac{d}{dx} \cos^{-1}\sqrt{x} + \cos^{-1}\sqrt{x} \frac{d}{dx} x$$

$$= x \times \frac{-1}{\sqrt{1-(\sqrt{x})^2}} \times \frac{1}{2\sqrt{x}} + \cos^{-1}\sqrt{x} \times 1$$

$$\boxed{\frac{dy}{dx} = \frac{-x}{2\sqrt{x}\sqrt{1-x}} + \cos^{-1}\sqrt{x}}$$

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11. i) $\cos^{-1}\left(\frac{1-x}{1+x}\right)$

$$y = \cos^{-1}\left(\frac{1-x}{1+x}\right)$$

$$= \frac{-1}{\sqrt{1-\left(\frac{1-x}{1+x}\right)^2}} \times \frac{d}{dx}\frac{1-x}{1+x}$$

$$= \frac{-(1+x)^2}{\sqrt{(1+x)^2-(1-x)^2}} \times \left[\frac{(1+x)(-1)-(1-x)}{(1+x)^2}\right]$$

$$= \frac{-1 \times (-1-x-1+x)}{(1+x)\sqrt{1+x^2+2x-(x+x^2-2x)}} = \frac{2}{(1+x)\sqrt{4x}} = \frac{2}{2(1+x)\sqrt{x}}$$ Ans

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$$\frac{d}{dx}\left(\frac{v}{u}\right) = \frac{u\frac{d}{dx}v - v\frac{d}{dx}u}{u^2}$$

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12. (i) $sin^{-1}x + sin^{-1}\sqrt{1-x^2}$

$y = \sin^{-1}x + \sin^{-1}\sqrt{1-x^2}$

$$\frac{dy}{dx} = \frac{d}{dx}\sin^{-1}x + \frac{d}{dx}\sin^{-1}\sqrt{1-x^2}$$

$$= \frac{1}{\sqrt{1-x^2}} + \frac{1}{\sqrt{1-(\sqrt{1-x^2})^2}}\frac{d}{dx}\sqrt{1-x^2}$$

$$= \frac{1}{\sqrt{1-x^2}} + \frac{1}{\sqrt{1-1+x^2}} \times \frac{1}{2\sqrt{1-x^2}} \times -2x$$

$$= \frac{1}{\sqrt{1-x^2}} + \frac{1}{x} \times \frac{-x}{\sqrt{1-x^2}} = 0$$

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15. i) if $y = \frac{\sin^{-1}x}{\sqrt{1-x^2}}$ prove that $(1-x^2)\frac{dy}{dx} - xy = 1$

$$\frac{dy}{dx} = \frac{d}{dx}\frac{\sin^{-1}x}{\sqrt{1-x^2}}$$

$$= \frac{\sqrt{1-x^2}\frac{d}{dx}\sin^{-1}x - \sin^{-1}x\frac{d}{dx}\sqrt{1-x^2}}{\left(\sqrt{1-x^2}\right)^2}$$

$$= \frac{\sqrt{1-x^2} \times \frac{1}{\sqrt{1-x^2}} - \sin^{-1}x \times \frac{1}{x\sqrt{1-x^2}} \times (-x)}{1-x^2}$$

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$$\frac{dy}{dx} = \frac{1 + \frac{x \sin^{-1}x}{\sqrt{1-x^2}}}{1-x^2}$$

$$(1-x^2)\frac{dy}{dx} = 1 + xy$$

$$\boxed{(1-x^2)\frac{dy}{dx} - xy = 1}$$

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ii) If $y = sin^{-1}\left(\frac{2x}{1+x^2}\right)$ Prove that $(1+x^2)\frac{dy}{dx}=2$

Let $x = \tan\theta$

$\theta = \tan^{-1}x$

$\sin 2x = \frac{2\tan x}{1+\tan^2 x}$

$y = \sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)$

$y = \sin^{-1}(\sin 2\theta)$

$y = 2\theta$

$y = 2\tan^{-1}x$

$\frac{dy}{dx} = 2\frac{d}{dx}\tan^{-1}x$

$\frac{dy}{dx} = 2 \times \frac{1}{1+x^2}$

$\frac{dy}{dx} = \frac{2}{1+x^2}$

$1+x^2 \frac{dy}{dx} = 2$

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16. i) $tan^{-1}\left(\frac{2x}{1-x^2}\right)$

$$y = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$$

$$y = \tan^{-1}\left(\frac{2\tan\theta}{1-\tan^2\theta}\right)$$

$$y = \tan^{-1}(\tan 2\theta)$$

$$y = 2\theta$$

$$y = 2\tan^{-1}x$$

$$\tan 2x = \frac{2\tan x}{1+\tan^2 x}$$

Let $x = \tan\theta$

$$\theta = \tan^{-1}x$$

$$\frac{dy}{dx} = 2\frac{d}{dx}\tan^{-1}x$$

$$\frac{dy}{dx} = 2 \times \frac{1}{1+x^2}$$

$$\boxed{\frac{dy}{dx} = \frac{2}{1+x^2}}$$

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